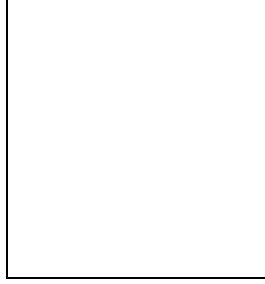


Why care about (θ_{13}, δ) degeneracy at future neutrino experiments

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The presence of several clone solutions in the simultaneous measurement of (θ_{13}, δ) has been widely discussed in literature. Here, after a pedagogical introduction on why these clones arise, we discuss how the clones location in the (θ_{13}, δ) plane change as a function of the physical input pair $(\bar{\theta}_{13}, \bar{\delta})$. We compare the clone flow of a set of possible future neutrino experiments: the CERN SuperBeam, BetaBeam and Neutrino Factory proposals. We show that the combination of these specific BetaBeam and SuperBeam could not help in solving the degeneracies. The combination of one of them with the Neutrino Factory Golden and Silver channel can, instead, be used to solve completely the eightfold degeneracy.

1 Introduction

The atmospheric and solar sector of the PMNS leptonic mixing matrix have been measured with quite good resolution by SK, SNO and KamLand. These experiments measure two angles, θ_{12} and θ_{23} , and two mass differences, Δm_{12}^2 and Δm_{23}^2 (for the explicit form of the PMNS matrix and the adopted conventions, see for example¹). The present bound on θ_{13} , $\sin^2 \theta_{13} \leq 0.02$, is extracted from the negative results of CHOOZ and from three-family analysis of atmospheric and solar data. The PMNS phase δ is totally unbounded as no experiment is sensitive to the leptonic CP violation. The main goal of next neutrino experiments will be to measure these two still unknown parameters. In this talk we concentrate on analyzing, from a theoretical point of view, the problem of degenerations that arise when trying to measure simultaneously (θ_{13}, δ) .

2 The Intrinsic Clone

In² it has been noticed that the appearance probability $P_{\alpha\beta}(\bar{\theta}_{13}, \bar{\delta})$ obtained for neutrinos at a fixed energy and baseline with input parameter $(\bar{\theta}_{13}, \bar{\delta})$ has no unique solution. Indeed, the equation:

$$P_{\alpha\beta}(\bar{\theta}_{13}, \bar{\delta}) = P_{\alpha\beta}(\theta_{13}, \delta) \quad (1)$$

has a continuous number of solutions. The locus of (θ_{13}, δ) satisfying this equation is called “equiprobability curve”, as can be seen in Fig. 1 (left). Considering the equiprobability curves for neutrinos and antineutrinos with the same energy (and the same input parameters), the

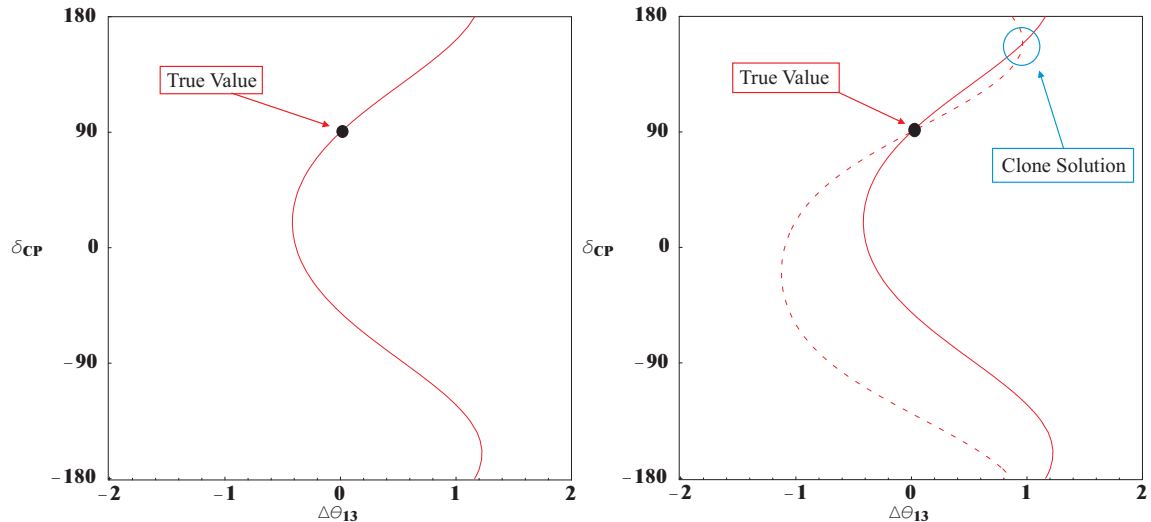


Figure 1: Clone solutions obtained solving the systems of Eq. 1 (left plot) and Eq. 2 (right plot).

system of equations (\pm referring to neutrinos and antineutrinos respectively):

$$P_{\alpha\beta}^{\pm}(\bar{\theta}_{13}, \bar{\delta}) = P_{\alpha\beta}^{\pm}(\theta_{13}, \delta) \quad (2)$$

has two intersections: the input pair $(\bar{\theta}_{13}, \bar{\delta})$ and a second, energy dependent, point. As can be noticed in Fig. 1 (right) this second intersection introduces an ambiguity in the measurement of the physical values of θ_{13} and δ : the so-called *intrinsic clone* solution.

Knowing the two probabilities of Eq. 2 is consequently not enough for solving the intrinsic degeneracy. One needs to add more information. Two ways are viable: i) using different “experiments” (same oscillation channel) and/or ii) using different oscillation “channels” (at the same experiment). In case i) one can think to observe the same neutrino oscillation channel (i.e. the golden $\nu_e \rightarrow \nu_\mu$ oscillation) at detectors located at different baseline L . In the left-side plot of Fig. 2 one can see that experiments at different L present clone solutions in different regions of the (θ_{13}, δ) parameter space. If the clones are well separated one can solve the degeneracy. The same result can be obtained if, for the specific neutrino flux considered, bins with quite different energy are available^a. Another possibility, case ii), is to send the neutrino flux towards only one facility, but to use contemporaneously two different oscillation channels (like for example $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$). In the right-side plot of Fig. 2 one can see how also in this case the intrinsic clones are expected in different locations and the degeneracy can be solved.

From this example we learn that the best way for solving the degeneracies is to add all the possible available information, i.e different baselines, different energy bins and different channels. Therefore, in planning future experiments one has to understand which combinations of experiments can give the largest set of (really) independent information. The existence of unresolved degeneracies could, in fact, manifests itself in a complete lost of predictability on the aforementioned parameters.

3 The Eightfold Degeneracy

Unfortunately, the appearance of the intrinsic degeneracy is only a part of the “clone problem”. As it was made clear in^{3,4} two other sources of ambiguities arise due to the present (and probably

^aLike for example in the case of the neutrino factory. This is an obvious consequence of the L/E dependence of the degeneracy position.

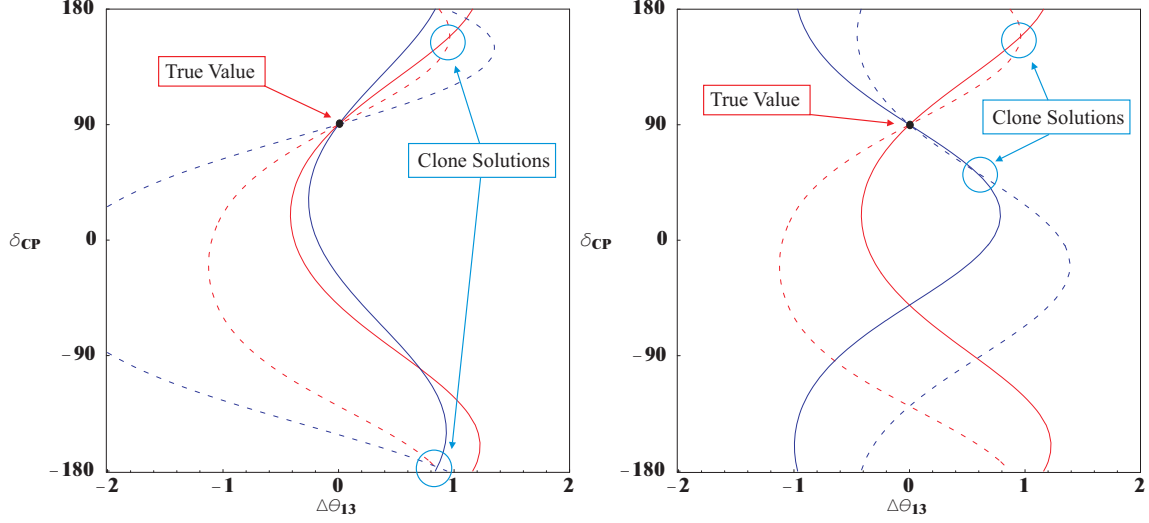


Figure 2: Solving the intrinsic degeneracy: two baseline $L=730$ and 3500 km, same channel example on the left, vs two channels $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$, same baseline example on the right.

near future) ignorance of the sign of the atmospheric mass difference, $s_{atm} = \text{sign}[\Delta m_{23}^2]$ and the θ_{23} octant, $s_{oct} = \text{sign}[\tan(2\theta_{23})]$. These two discrete variables assume the values ± 1 , depending on the physical assignments of the Δm_{23}^2 sign ($s_{atm} = 1$ for $m_3^2 > m_2^2$ and $s_{atm} = -1$ for $m_3^2 < m_2^2$) and of the θ_{23} octant ($s_{oct} = 1$ for $\theta_{23} < \pi/4$ and $s_{oct} = -1$ for $\theta_{23} > \pi/4$). As a consequence, future experiments will have as ultimate goal the measure of the two continuous variables θ_{13} and δ plus the two discrete variables s_{atm} and s_{oct} .

Before following with the analysis of the clones position, it should be noticed that experimental results are not given in terms of oscillation probabilities but of number of charged leptons observed in a specific detector. We must therefore integrate the oscillation probability over the neutrino flux, the νN cross-section and the detector efficiency $\epsilon(E_\mu)$. Eq. (2) is thus replaced by

$$N_\beta^\pm(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct}) = N_\beta^\pm(\theta_{13}, \delta; s_{atm} = \bar{s}_{atm}; s_{oct} = \bar{s}_{oct}), \quad (3)$$

with β the lepton flavour corresponding to the oscillated neutrino. In Eq. (3) we have implicitly assumed to know the right sign and the right octant for the atmospheric mass difference and angle. As these quantities are unknown (and they will remain so in the near future) the following systems of equations should be considered as well:

$$N_\beta^\pm(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct}) = N_\beta^\pm(\theta_{13}, \delta; s_{atm} = -\bar{s}_{atm}, s_{oct} = \bar{s}_{oct}) \quad (4)$$

$$N_\beta^\pm(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct}) = N_\beta^\pm(\theta_{13}, \delta; s_{atm} = \bar{s}_{atm}, s_{oct} = -\bar{s}_{oct}) \quad (5)$$

$$N_\beta^\pm(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct}) = N_\beta^\pm(\theta_{13}, \delta; s_{atm} = -\bar{s}_{atm}, s_{oct} = -\bar{s}_{oct}) \quad (6)$$

Solving the four systems of Eqs. (3)-(6) will result in obtaining the true solution plus the appearance of additional *clones* to form an eightfold-degeneracy⁴. These eight solutions are:

- the true solution and its *intrinsic clone*, obtained solving the system of Eq. (3);
- the Δm_{23}^2 -sign clones (hereafter called *sign clones*) of the true and intrinsic solution, obtained solving the system of Eq. (4);
- the θ_{23} -octant clones (hereafter called *octant clones*) of the true and intrinsic solution, obtained solving the system of Eq. (5);

- the Δm_{atm}^2 -sign θ_{23} -octant clones (hereafter called *mixed clones*) of the true and intrinsic solution, obtained solving the system of Eq. (6).

It has been noticed⁵ that clones location calculated starting from the probability or the number of events can be significantly different.

4 The Degeneracies flows

The analytical description of the clones flows is beyond the non-expert cut of this talk and can be found in⁵. Here we will apply those results to illustrate with a specific example, how studying the clones location can give us a hint on which combination of experiments is better suited to solve some (or all) of the degeneracies. It must be noticed that this analysis only provides a useful tool to detect the experiment synergies that are most promising to measure these two still unknown entries of the PMNS matrix. These results must then be confirmed by a detailed analysis in which statistics and systematics of a given experiment combination are carefully taken into account.

The experiments that we have considered in our analysis are the following:

- A Neutrino Factory with 50 GeV muons circulating in the storage ring and two detectors: the first one located at $L = 2810$ Km is designed to look for the golden channel (hereafter labeled as NFG); the second one located at $L = 732$ Km is designed to look for the silver channel (hereafter labeled as NFS). The average neutrino energy is: $\langle E_{\nu_e, \bar{\nu}_e} \rangle \simeq 30$ GeV.
- A SuperBeam with 2 GeV protons and one detector located at $L = 130$ Km to look for $\nu_\mu \rightarrow \nu_e$ (hereafter labeled as SB). The average neutrino energy is: $\langle E_{\nu_\mu} \rangle \simeq 270$ MeV and $\langle E_{\bar{\nu}_\mu} \rangle \simeq 250$ MeV.
- A Beta Beam with ${}^6\text{He}$ and ${}^{18}\text{Ne}$ ions and one detector located at $L = 130$ Km to look for $\nu_e \rightarrow \nu_\mu$ (hereafter labeled as BB). The average neutrino energy is: $\langle E_{\nu_e} \rangle \simeq 370$ MeV and $\langle E_{\bar{\nu}_e} \rangle \simeq 230$ MeV.

Also if for our theoretical analysis the details of the detectors (such as the mass or the specific technology adopted to look for a given signal) are not fundamentals, the relevant parameters (the neutrino fluxes and the baseline) have been chosen considering the following proposed experiments: (NFG)^{6,7}, (NFS)^{8,9}, (SB)¹⁰ and (BB)¹¹. For the different beams we have taken as representative the CERN Neutrino Factory, SPL and BetaBeam proposals⁶.

The theoretical requirement for solving ambiguities is to find a specific combination of experiments such that the corresponding clone flows lie “well apart”. In this scenario an experimental fit to the data will result in a good χ^2 absolute minimum near the true solution, where the information from different experiments adds coherently. Near the clone locations poor χ^2 relative minima will be found, since here the results of different experiments do not add coherently.

In the following subsections we analyze the flows of the different degeneracies for the four considered facilities. We compute the theoretical clone locations for a variable $\bar{\theta}_{13}$ in the range $\bar{\theta}_{13} \in [0.1^\circ, 10^\circ]$, going from the present CHOOZ upper bound $[\sin^2 2\bar{\theta}_{13}]_{max} = \mathcal{O}(10^{-1})$ down to $[\sin^2 2\bar{\theta}_{13}]_{min} = \mathcal{O}(10^{-5})$. We show here due to the limited space available only the $\bar{\delta} = 90^\circ$ case corresponding to the maximal leptonic CP-violation phase.

^bSee for example in¹² for a detailed description of each of these proposals and¹³ for a description of a higher energy BB proposal.

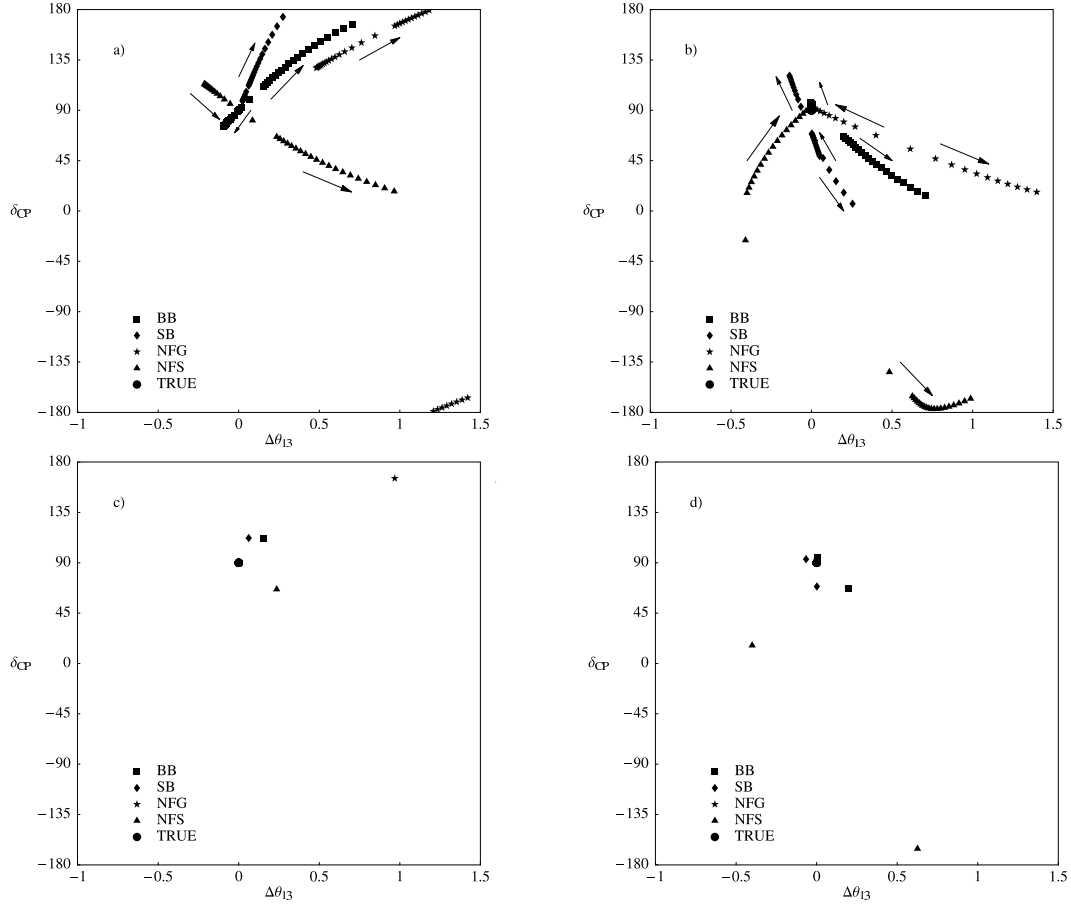


Figure 3: Clone location in the $(\Delta\theta_{13}, \delta)$ plane for the intrinsic (left) and sign (right) degeneracies: BB (boxes); SB (diamonds); NFG (stars) and NFS (triangles). The full circle is the true solution. (a) and (b) represent the intrinsic and sign clone flows for $\bar{\theta}_{13} \in [0.1^\circ, 10^\circ]$ and $\bar{\delta} = 90^\circ$; (c) and (d) represent the case $\bar{\theta}_{13} = 1^\circ$.

4.1 The Intrinsic and the Sign Clone Flow

In Fig. 3 we present the clone flow for the intrinsic (left) and the sign (right) degeneracies. The clone locations have been computed for $\bar{\theta}_{13}$ in the range $\bar{\theta}_{13} \in [0.1^\circ, 10^\circ]$ and $\bar{\delta} = 90^\circ$ (upper plots) or for $\bar{\theta}_{13} = 1^\circ, \bar{\delta} = 90^\circ$ (lower plots), on the verge of the sensitivity limit. The arrows indicate the direction of the flows from large to small $\bar{\theta}_{13}$. In each figure the results for the four facilities are presented together, in order to show how the combination of two or more of them can help solving that specific degeneracy. As it can be seen the BB and SB clones both for the intrinsic and the sign degeneracy lies very near, making these two experiments not the best choice for resolving these degeneracies even if in the specific case shown in the lower plots, the loss of predictivity on θ_{13} and δ seems “acceptable”. Conversely NFG or NFS clones lie in a well separated region. The golden channel combined with the silver and/or the SB/BB facilities is always, in principle, capable to solve the intrinsic ambiguity, while the golden channel alone can solve the sign ambiguity (at least for $\theta_{13} \geq 1^\circ$).

4.2 The Octant and the Mixed Clones

If θ_{23} were maximal no additional degeneracies would be present. Otherwise two further degeneracies do appear. In Fig. 4 we present the clone flow for the octant (left) and the mixed (right) degeneracies. The clone locations have been computed for $\bar{\theta}_{13}$ in the range $\bar{\theta}_{13} \in [0.1^\circ, 10^\circ]$ and $\bar{\delta} = 90^\circ$ (upper plot) or for a fixed value, $\bar{\theta}_{13} = 1^\circ, \bar{\delta} = 90^\circ$ (lower plots). In each figure the results for the four facilities are presented together. Again we notice that the BB and SB flows for the octant and mixed degeneracies lie always very near making difficult the task of solving degeneracies using only these two facilities. This seems a general consequence of the quite similar

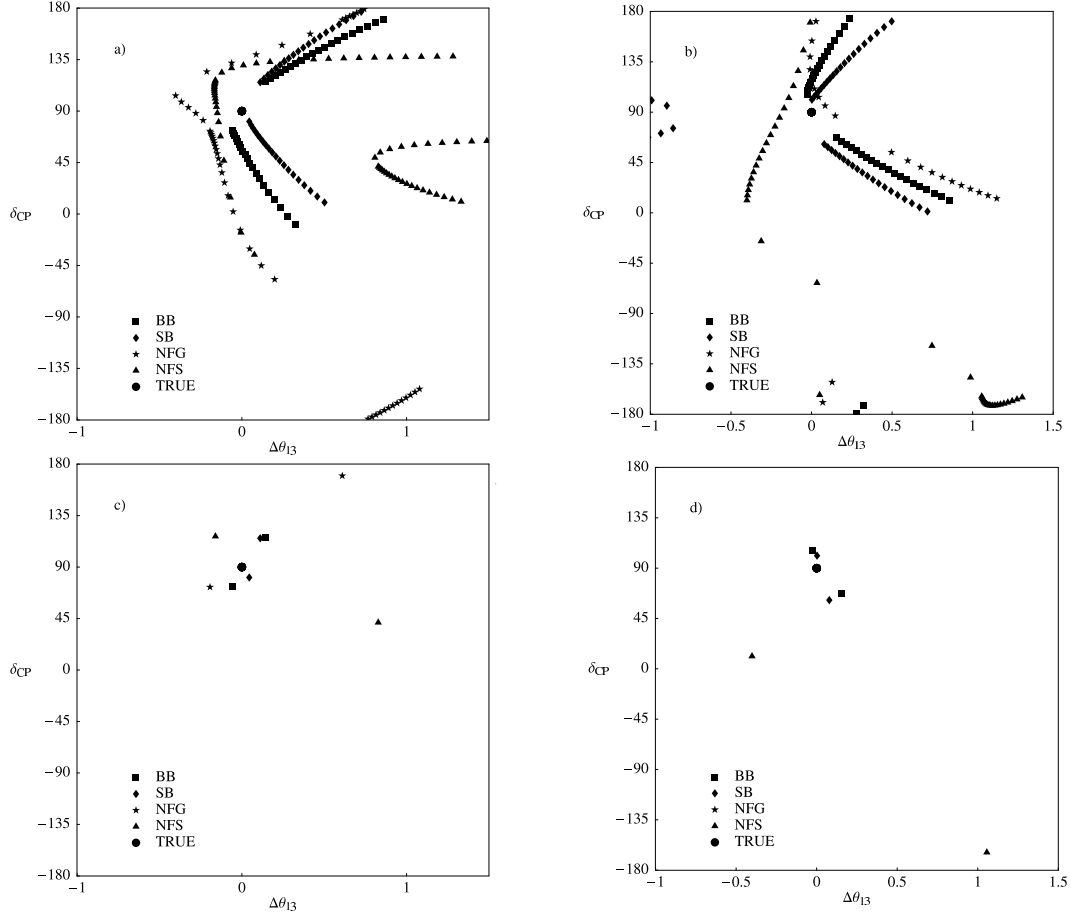


Figure 4: Clone location in the $(\Delta\theta_{13}, \delta)$ plane for the octant (left) and mixed (right) degeneracies: BB (boxes); SB (diamonds); NFG (stars) and NFS (triangles). The full circle is the true solution. (a) and (b) represent the octant and mixed clone flows for $\theta_{13} \in [0.1^\circ, 10^\circ]$ and $\delta = 90^\circ$; (c) and (d) represent the case $\theta_{13} = 1^\circ$. The results have been computed with $\theta_{23} = 40^\circ$.

beam design (baseline and flux). NFG and NFS flows are instead always well separated except for particularly small θ_{13} values when some superposition can be observed. Anyway using the Neutrino Factory plus on of the SB/BB seems possible, in principle, to solve all the degeneracies. NFG alone can solve the mixed degeneracy being sensitive to the sign of Δm_{23}^2 (at least for $\theta_{13} \geq 1^\circ$).

5 Conclusions

In this talk we have analyzed, from a theoretical point of view, the problem of degenerations that arise when trying to measure simultaneously (θ_{13}, δ) . The existence of unresolved degeneracies could, in fact, manifests itself in a complete loss of predictability on the aforementioned parameters at future neutrino facilities. Therefore, in planning future experiments will be important to understand which combinations of experiments can give the largest set of (really) independent information. With the setup considered we noticed that BetaBeam/SuperBeam combination seems less effective than the Neutrino Factory/BetaBeam (SuperBeam) combination to solve the eightfold degeneracy.

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